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Strong magnetic limit of string theory

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Abstract: We show that there exists a certain limit in type-I and type-II superstring theory in the presence of a suitable configuration of magnetic U(1) fields where all string excitations get an infinite mass, except for the neutral massless sector and for the boson and fermion string states lying on the leading Regge trajectory. For a supersymmetric configuration of magnetic fields in internal directions, the resulting theory after the limit is a 3+1 Lorentz invariant supersymmetric theory. Supersymmetry can be broken by introducing extra components of the magnetic field or else by finite temperature. In both cases we compute the one-loop partition function for the type-I string model after taking the limit, which turns out to be different from the Yang-Mills result that arises by a direct $\alpha' \to 0$ limit. In the case of finite temperature, no Hagedorn transition appears, in consistency with the reduction of the string spectrum. In type-II superstring theory, the analogous limit is constructed by starting with a configuration of Melvin twists in two or more complex planes. The resulting theory contains gravitation plus the infinite number of states of the leading Regge trajectory.

Keywords: Conformal Field Models in String Theory, D-branes.

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1. Introduction

Taking limits in string theory have led to insightful connections. One important limit is the low energy limit represented by $\alpha' \to 0$, leading to Yang-Mills theory and supergravity [1]. In this limit, all superstring excitations become infinitely massive and decouple, leaving only the massless multiplet. Another important limit is the one connecting conformal field theories to string theory in AdS backgrounds [2]. The limit is again a low energy $\alpha' \to 0$ limit, but this time massive string excitations remain due to the fact that the limit is taken in a way that one is simultaneously approaching the horizon region where there is an infinite redshift, leading to string excitations of finite mass. Furthermore, by taking a suitable limit of string theory it was shown in [3] that non-commutative field theories exist as quantum field theories.

In the present work we will show that there exists an $\alpha' \to 0$ limit where string theory gets greatly simplified, due to the fact that most massive excitations decouple. The resulting theory includes in the spectrum Yang-Mills vector fields (in the type-I case), the supergravity multiplet (in the type-II case), and an infinite tower of states corresponding to string states lying on the leading Regge trajectory. The number of states surviving at each mass level N is proportional to N. This is a very small number, compared to the exponential $\sim \exp[\mathrm{const.}\sqrt{N}]$ number of states at level N in the usual string theory spectrum.

Preserving a part of the string spectrum in the limit $\alpha' \to 0$ requires adding interactions which can lower the energy of a string state up to zero. This can be achieved by introducing magnetic fields in some directions. Due to the gyromagnetic coupling, the magnetic field lowers the energy of a string state with the spin aligned with the magnetic field direction (open strings in magnetic fields have been first considered in [4–6]). This typically leads to

Nielsen-Olesen [7] tachyons for critical values of the magnetic fields [8, 9] (similar tachyons in closed string magnetic models are discussed in [10, 11] and references therein). Here we shall avoid tachyons by starting with a supersymmetric configuration of magnetic fields. In this case, we will see that at a critical magnetic field (which is infinite in the type-I string model and finite in the type-II string model) the states of the leading Regge trajectory with spins aligned with the magnetic field become massless. By taking a simultaneous limit with $\alpha' \to 0$, so that the masses of these states remain finite and non-zero, the masses of all other string states go to infinity, except for the neutral massless string states, which remain massless.

This paper is organized as follows. The models related to type-I superstring theory are constructed in section 2. In section 3 we study the partition function for non-supersymmetric models and for the supersymmetric model at finite temperature. In the case of finite temperature, one finds a finite expression for all β, which in particular shows the absence of a Hagedorn transition. Finally, in section 4 we construct the type-II model. We show that the resulting theory after the limit contains the type-II massless supergravity multiplet plus and infinite number of states with maximum angular momentum.

2. The string model

2.1 Superstring spectrum and limit

We first consider type-I superstring theory in the presence of two magnetic fields F_{45} and F_{67}.[1] The superstring spectrum is given by

$$\begin{aligned} \alpha' M^2 &= \hat{N} - \varphi_1 \hat{J}_1 - \varphi_2 \hat{J}_2 \,, && (2.1)\\ \hat{J}_1 &= S_1 - l_1 - \frac{1}{2}\,, \qquad \hat{J}_2 = S_2 - l_2 - \frac{1}{2}\,, && (2.2)\\ \pi\varphi_1 &= \arctan 2\pi\alpha' e F_{45} + \arctan 2\pi\alpha' e' F_{45}\,, && \\ \pi\varphi_2 &= \arctan 2\pi\alpha' e F_{67} + \arctan 2\pi\alpha' e' F_{67}\,, && (2.3)\\ 0 &< \varphi_{1,2} < 1\,. && (2.4) \end{aligned}$$

The indices 1 and 2 refer to the directions 45 and 67 respectively, whereas e and e' are the electric U(1) charges at the two endpoints of the open string. Here $\hat{N}$ includes the normal ordering constant, so that $\hat{N} = 0, 1, 2, \ldots$. The Landau numbers take value $l_{1,2} = 0, 1, 2, \ldots$. The spin quantum numbers of a given state obey the inequality

$$|S_1 \pm S_2| \le \hat{N} + 1\,. \qquad (2.5)$$

For generic values of φ_1 and φ_2, the spectrum exhibits [8, 9] magnetic instabilities of Nielsen-Olesen type [7]. States with maximum angular momentum aligned with the

[1] In realistic compactifications, the directions 4,5,6,7,8,9 can be compact so that the string model is Lorentz invariant in 3+1 dimensions. Models with supersymmetry breaking due to internal magnetic fields were constructed in [12].

magnetic field become tachyonic above some critical value of the magnetic field. As an example, consider the following state (in the light-cone gauge):

$$|\Phi_0\rangle = (\alpha_{-1}^{(4)} + i\alpha_{-1}^{(5)})^n (b_{-1/2}^{(4)} + ib_{-1/2}^{(5)})\ |0\rangle_{\rm NS}\,, \tag{2.6}$$

where α_n^μ are as usual the mode operators of X^μ, and b_r^μ are the mode operators of ψ^μ in the NS sector. This state has mass

$$\alpha' M^2 = n(1-\varphi_1) - \frac{(\varphi_1 - \varphi_2)}{2}\,. \tag{2.7}$$

It becomes tachyonic for all n with $n < (\varphi_1 - \varphi_2)/(2(1-\varphi_1))$, $\varphi_1 > \varphi_2$.

If $\varphi_1 = \varphi_2 \equiv \varphi$, all states have $M^2 \geq 0$. The reason is that this special configuration is supersymmetric (preserving 1/2 of the original 16 supersymmetries). Now consider this configuration and states with $l_1 = l_2 = 0$, $S_1 + S_2 = \hat{N} + 1$, so that $\hat{J}_1 + \hat{J}_2 = \hat{N}$. For such states, the mass spectrum (2.1) becomes

$$\alpha' M^2 = \hat{N}(1-\varphi)\,. \tag{2.8}$$

Next, we write $\alpha' = \epsilon \alpha'_{\rm eff}$ and $\varphi = 1 - \epsilon$, and take the limit $\epsilon \to 0$. This limit corresponds to sending $\alpha' F_{45} = \alpha' F_{67}$ to ∞,

$$2\pi\alpha' F_{45} \to \frac{e+e'}{\pi e e'}\,\frac{1}{\epsilon}\,.$$

The limit is similar to the limit of [3] that leads to non-commutative super Yang-Mills theory. In the present case, it is important that there are two components $F_{45} = F_{67}$. Also, the open string of [3] has vanishing total charge, $e + e' = 0$, whereas here we consider both sectors, charged and neutral open strings.

The mass spectrum (2.8) for the states with $\hat{J}_1 + \hat{J}_2 = \hat{N}$ reduces to

$$\alpha'_{\rm eff} M^2 = \hat{N}\,, \tag{2.9}$$

whereas for any other state in the spectrum one has $\hat{J}_1 + \hat{J}_2 < \hat{N}$ so that

$$\alpha'_{\rm eff} M^2_{\rm other} = \frac{1}{\epsilon}(\hat{N} - \hat{J}_1 - \hat{J}_2)\ \to\ \infty\,. \tag{2.10}$$

Thus in the simultaneous limit $\varphi \to 1$ and $\alpha' \to 0$, the states with maximum angular momentum aligned with the magnetic field remain with finite mass and all other string states in the spectrum decouple.

Consider now the neutral sector with $e + e' = 0$. Here the mass spectrum is not affected by the magnetic field,

$$\alpha' M^2 = \hat{N}\,. \tag{2.11}$$

In the limit $\epsilon \to 0$, all states are decoupled except for the massless states with $\hat{N} = 0$. Altogether, the full theory after the limit contains a massless gauge supermultiplet which is neutral under the U(1) F field turned on in the 45 and 67 directions, plus an infinite number of states of the leading Regge trajectory. This infinite set of states also includes charged vector fields corresponding to $\hat{N} = 0$ level having $S_1 = 1$, $S_2 = 0$ or $S_1 = 0$, $S_2 = 1$.

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Explicitly, surviving charged states are as follows:
NS sector:

$$|\Phi^{I}_{\rm boson}\rangle = (\alpha^{(4)}_{-1} + i\alpha^{(5)}_{-1})^{n_1}(\alpha^{(6)}_{-1} + i\alpha^{(7)}_{-1})^{n_2}(b^{(4)}_{-1/2} + ib^{(5)}_{-1/2})\ |0\rangle_{\rm NS} \tag{2.12}$$

$$|\Phi^{II}_{\rm boson}\rangle = (\alpha^{(4)}_{-1} + i\alpha^{(5)}_{-1})^{n_1}(\alpha^{(6)}_{-1} + i\alpha^{(7)}_{-1})^{n_2}(b^{(6)}_{-1/2} + ib^{(7)}_{-1/2})\ |0\rangle_{\rm NS} \tag{2.13}$$

with $n_1 + n_2 = \hat{N}$. These are $2(\hat{N}+1)$ bosonic states, with $\hat{N} = 0, 1, 2, \ldots$. There are also surviving states involving three $b^{\mu}_{-1/2}$ operators, as well as all fermion superpartners coming from the R sector.

2.2 Non-supersymmetric string models

One can break supersymmetry by introducing another magnetic field component F_{89}. The resulting spectrum is now

$$\alpha' M^2 = \hat{N} - \varphi(\hat{J}_1 + \hat{J}_2) - \varphi_3 \hat{J}_3\,, \tag{2.14}$$

$$\pi\varphi_3 = \arctan 2\pi\alpha' e F_{89} + \arctan 2\pi\alpha' e' F_{89}\,, \tag{2.15}$$

with $\hat{J}_3 \equiv \hat{J}_{89}$. Now we scale the parameters as follows

$$\alpha' = \epsilon\ \alpha'_{\rm eff}\,, \qquad \varphi_3 = \epsilon\ \bar{\varphi}_3\,, \qquad \varphi = 1 - \epsilon\,, \tag{2.16}$$

and consider the limit $\epsilon \to 0$ with $\alpha'_{\rm eff}$ and $\bar{\varphi}_3$ fixed. As a result, only states with maximum spin aligned with the magnetic field remain of finite mass, $S_1 + S_2 = \hat{N} + 1$, or $\hat{J}_1 + \hat{J}_2 = \hat{N}$, just as in the previous case. All other charged string states are decoupled. The masses of the surviving states are now given by

$$\alpha'_{\rm eff} M^2 = \hat{N} - \bar{\varphi}_3 \hat{J}_3\,. \tag{2.17}$$

There is a splitting between fermion and boson masses due to the gyromagnetic coupling with the magnetic field $\bar{\varphi}_3$. In the NS sector, $\hat{J}_3 = -l_3 - \frac{1}{2}$, $l_3 = 0, 1, 2, \ldots$, since the requirement $S_1 + S_2 = \hat{N} + 1$ implies that $S_3 = 0$. This shows that this model has no tachyons.

A different non-supersymmetric model can be obtained as follows. One consider the model with φ_1 and φ_2 and write

$$\alpha' = \epsilon\, \alpha'_{\rm eff}\,, \qquad \varphi_1 = 1 - a_1\epsilon\,, \qquad \varphi_2 = 1 - a_2\epsilon\,. \tag{2.18}$$

As $\epsilon \to 0$ with $\alpha'_{\rm eff}$, a_1, a_2 fixed, only the states with $\hat{J}_1 + \hat{J}_2 = \hat{N}$ survive, just as before. Their mass formula is given by

$$\alpha'_{\rm eff} M^2 = a_1 \hat{J}_1 + a_2 \hat{J}_2\,, \qquad \hat{J}_1 + \hat{J}_2 = \hat{N}\,. \tag{2.19}$$

For $a_1 = a_2$ we recover the supersymmetric case. The model is characterized by two independent parameters, say $\alpha'_{\rm eff}$ and a_2, since the other parameter, a_1, can be absorbed into a redefinition of the $\alpha'_{\rm eff}$, a_2.

Unlike the previous non-supersymmetric model (2.17), this model has tachyons appearing for $\hat{J}_1 = -\frac{1}{2}$, $\hat{J}_2 = \hat{N} + \frac{1}{2}$ or $\hat{J}_2 = -\frac{1}{2}$, $\hat{J}_1 = \hat{N} + \frac{1}{2}$, with $\hat{N} < (a_1 - a_2)/(2a_2)$ for $a_1 > a_2$. They will be reflected as IR divergences in the one-loop partition function.

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3. Partition function

As a further test of the model that results upon taking the limit of section 2, in this section we compute the one-loop partition function. This can be obtained by taking the limit of the partition function for open superstring theory in the presence of several magnetic fields (at zero and finite temperature) computed by Tseytlin in [13].

3.1 Non-supersymmetric models at zero temperature

We first consider the string model with $\varphi \equiv \varphi_1 = \varphi_2$ and φ_3. In the present case, the partition function of [13] takes the form

$$Z = c_1 \int_0^\infty \frac{dt}{t^2} \prod_{I=1}^{4} \frac{f_I}{\theta_1(it\varphi_I|it)} \left[\prod_{J=1}^{4} \theta_2(it\varphi_J|it) - \prod_{J=1}^{4} \theta_3(it\varphi_J|it) + \prod_{J=1}^{4} \theta_4(it\varphi_J|it) \right] \tag{3.1}$$
$$f \equiv f_1 = f_2 = 2\pi\alpha' F_{45}(e+e')\,, \qquad f_3 = 2\pi\alpha' F_{89}(e+e')\,, \qquad \varphi_4 = 0\,.$$

Now we rescale parameters as follows:

$$\alpha' = \epsilon\, \alpha'_{\rm eff}\,, \qquad \varphi = 1-\epsilon\,, \qquad \varphi_3 = \epsilon\bar\varphi_3\,, \qquad t = \frac{\rm t}{\epsilon}\,. \tag{3.2}$$

Taking the limit $\epsilon \to 0$ with $\alpha'_{\rm eff}$, $\bar\varphi_3$ fixed, the partition function becomes

$$Z = \epsilon c_1 f_0^2 \pi\bar\varphi_3 \int_0^\infty \frac{d\rm t}{\rm t^3} \tanh\left(\frac{\pi\bar\varphi_3 \rm t}{2}\right) \frac{\left(1 - 2e^{-2\pi \rm t}\cosh(\pi\bar\varphi_3 \rm t) + e^{-4\pi \rm t}\right)}{(1-e^{-2\pi \rm t})^2}\,, \qquad f_0 = \frac{(e+e')^2}{\pi\, ee'}\,. \tag{3.3}$$

For $\bar\varphi_3 = 0$, the partition function vanishes identically, as expected, since the model becomes supersymmetric. Note the overall ϵ factor.

The IR region is at $\rm t \to \infty$. The integral becomes $\int^\infty d{\rm t}/{\rm t}^3$ which is convergent. In the UV region, $\rm t \to 0$, and we find that the integral diverges as $\int_0 d{\rm t}/{\rm t}^2$.

The result (3.3) is very different from what is obtained by a direct $\alpha' \to 0$ limit of (3.1). In this limit only the Yang-Mills supermultiplet survives, and –as observed in [13]– one finds a result which is proportional to the $d = 10$ Yang-Mills one-loop effective action. In the case of SU(2) theory with a U(1) background field $\hat F_{\mu\nu} = \sigma_3 F_{\mu\nu}$, with $F_{45} = f$, $F_{67} = f$ and $F_{89} = f_3$, the one-loop effective action is [14, 15]

$$\Gamma_{\rm YM} = \int_\delta^\infty \frac{d\rm t}{\rm t^3} \frac{f^2 f_3}{\sinh^2(f{\rm t})\sinh(f_3{\rm t})} \left(2\cosh(2f{\rm t}) + \cosh(2f_3{\rm t}) + 1 - 4\cosh^2(f{\rm t})\cosh(f_3{\rm t})\right). \tag{3.4}$$

In the UV $\rm t \to 0$ region, the integral behaves as $\int_0 d{\rm t}/{\rm t}^2$, just as (3.3).

Now consider the partition function of the non-supersymmetric model with two magnetic fields and mass spectrum given by (2.19). We rescale parameters as follows

$$\alpha' = \epsilon\, \alpha'_{\rm eff}\,, \qquad \varphi_1 = 1 - a_1\epsilon\,, \qquad \varphi_2 = 1 - a_2\epsilon\,, \qquad t = \frac{\rm t}{\epsilon}\,. \tag{3.5}$$

By taking the $\epsilon \to 0$ limit in (3.1), we obtain

$$Z = \frac{1}{2}\epsilon c_1 f_0^2 \int_0^\infty \frac{d\rm t}{\rm t^4} \frac{\left(\cosh(a_1\pi{\rm t}) - \cosh(a_2\pi{\rm t})\right)^2}{\sinh(a_1\pi t)\sinh(a_2\pi t)}\,. \tag{3.6}$$

In the UV t $\to 0$ region, the integral behaves as $\int_0 \mathrm{dt}/\mathrm{t}^2$, as in the previous model. In the IR t $\to \infty$ region, the integral is divergent, $\int^\infty \mathrm{dt}/\mathrm{t}^4 e^{\pi(a_2-a_1)\mathrm{t}}$, for $a_2 > a_1$. This is due to the presence of tachyons in this particular model as pointed out in section 2.

3.2 Supersymmetric model at finite temperature

Now we consider the model $\varphi \equiv \varphi_1 = \varphi_2$ at finite temperature $T = \beta^{-1}$. The resulting model after taking the limit $\varphi \to 1$ depends on three parameters α'_{eff}, β and the string coupling g_s.

The string-theory partition function before the limit is given by [13]

$$Z = a_1\beta \int_0^\infty \frac{dt}{t^2}\,\theta_2\Big(0|\frac{i\beta^2}{2\pi^2\alpha' t}\Big) \prod_{I=1}^{4} f_I \frac{\theta_2(it\hat{\varphi}_I|it)}{\theta_1(it\varphi_I|it)}\,. \tag{3.7}$$

In the present case, $\varphi_1 = \varphi_2 \equiv \varphi$, $\varphi_3 = \varphi_4 = 0$, $\hat{\varphi}_1 = \hat{\varphi}_2 = 0$, $\hat{\varphi}_3 = \hat{\varphi}_4 = \varphi$.

Now rescale variables as follows

$$\alpha' = \epsilon\,\alpha'_{\text{eff}}\,, \qquad \varphi = 1-\epsilon\,, \qquad t = \frac{\mathrm{t}}{\epsilon}\,. \tag{3.8}$$

After taking the limit $\epsilon \to 0$ with fixed α'_{eff}, β, we find the following partition function

$$Z = \epsilon a_1 f_0^2 \beta \int_0^\infty \frac{\mathrm{dt}}{\mathrm{t}^4}\,\theta_2\Big(0|\frac{i\beta^2}{2\pi^2\alpha'_{\text{eff}}\mathrm{t}}\Big)\frac{(1+e^{-2\pi\mathrm{t}})^2}{(1-e^{-2\pi\mathrm{t}})^2}\,. \tag{3.9}$$

This is in agreement with the general expression for the free energy in $p+1$ dimensional field theory with mass operator $\hat{M} = \hat{M}_B = \hat{M}_F$,

$$Z = V_p\beta \int_0^\infty \frac{\mathrm{dt}}{\mathrm{t}^{\frac{p+3}{2}}}\theta_2\Big(0|\frac{i\beta^2}{2\pi^2\alpha'_{\text{eff}}\mathrm{t}}\Big)\,\mathrm{Tr}\, e^{-\mathrm{t}M^2}\,. \tag{3.10}$$

To see this, we note that, in the present case, the boson and fermion mass spectrum of section 2 is given by $\alpha'_{\text{eff}}\hat{M}^2 = \hat{N}$, with $\hat{J}_1 + \hat{J}_2 = \hat{N}$, and $p = 5$ is the number of spatial dimensions with translational invariance (i.e. x^μ with $\mu = 1,2,3,8,9$).

Let us now examine the convergence properties of the thermal partition function.

- In the UV limit, t $\to 0$ and $\theta_2\big(0|\frac{i\beta^2}{2\pi^2\alpha'_{\text{eff}}\mathrm{t}}\big) \to 2\exp\big[-\frac{\beta^2}{8\pi\alpha'_{\text{eff}}\mathrm{t}}\big]$ and the integral is convergent for any value of β. This shows that there is no Hagedorn critical temperature. This is expected, since the Hagedorn temperature is due to the rapid growth of the density of string states with the mass. In the present case, most of the string states have decoupled. The growth of the number of surviving string states is only linear with $\hat{N}$ and as a result no Hagedorn transition appears.

- In the IR region, t $\to \infty$. Using the modular transformation $\theta_2(0|\frac{i}{\tau}) = \sqrt{\tau}\theta_4(0|i\tau)$, and the fact that $\theta_4(0|i\tau) \to 1$ as $\tau \to \infty$, we find that the integral (3.9) takes the form $\int^\infty \mathrm{dt}/\mathrm{t}^{7/2}$, which is convergent at infinity.

4. Type-II superstrings

A simple model of type-II string theory in a magnetic background is the one considered in [10], where the magnetic field is of Kaluza-Klein origin. This has been generalized to include several magnetic field components in [11] and in [16]. We follow the notation of [11]. The bosonic part of the GS lagrangian is

$$\begin{aligned} L_B &= \partial_+ x_i \partial_- x_i + (\partial_+ + ib_1\partial_+ y) z_1 (\partial_- - ib_1 \partial_- y) z_1^* + \\ &\quad + (\partial_+ + ib_2\partial_+ y) z_2 (\partial_- - ib_2 \partial_- y) z_2^* + \partial_+ y \partial_- y \,, \\ z_1 &= x_4 + ix_5 \,, \qquad z_2 = x_6 + ix_7 \,. \end{aligned} \tag{4.1}$$

Here y is a compact coordinate with $y = y + 2\pi R$. This is an exact CFT. This is clear from the fact that the σ-model metric describes Minkowski space with some identifications, so the Riemann tensor is identically zero. The dimensional reduction in y gives a magnetic background with curvature proportional to the electromagnetic stress tensor. Thus taking a strong magnetic limit will imply strong curvatures for the dimensionally reduced metric.

We consider a configuration with $b_1 = b_2$, which preserves 1/2 of the original 32 supersymmetries [11, 16]. The superstring spectrum is given by

$$\begin{aligned} \alpha' M^2 &= 2\big(\hat N_R - \hat\gamma(\hat J_{1R} + \hat J_{2R})\big) + 2\big(\hat N_L + \hat\gamma(\hat J_{1L} + \hat J_{2L})\big) + \\ &\quad + \frac{\alpha'}{R^2}\big(m - bR(\hat J_1 + \hat J_2)\big)^2 + \frac{w^2 R^2}{\alpha'} \end{aligned} \tag{4.2}$$

$$\hat N_R - \hat N_L = mw \,, \tag{4.3}$$

$$\hat J_s = \hat J_{sL} + \hat J_{sR} \,, \quad \hat J_{sL} = S_{sL} + l_{sL} + \frac{1}{2} \,, \quad \hat J_{sR} = S_{sR} - l_{sR} - \frac{1}{2} \,, \qquad s = 1, 2 \,, \tag{4.4}$$

$$l_{sL} \,,\ l_{sR} = 0, 1, 2, \dots$$

where m and w are, respectively, Kaluza-Klein momentum and winding numbers around the compact direction y, $\hat\gamma \equiv \gamma - [\gamma]$, $\gamma = wbR$, and $[\gamma]$ is the integer part of γ.

Now we write

$$\alpha' = \epsilon\, \alpha'_{\rm eff} \,, \qquad bR = 1 - \epsilon \,, \qquad R = \epsilon \bar R \,, \tag{4.5}$$

so that $\hat\gamma = 1 - w\epsilon$ for $w \neq 0$. We find

$$\begin{aligned} \alpha'_{\rm eff} M^2 &= \frac{2}{\epsilon}\big(\hat N_R - (1-w\epsilon)(\hat J_{1R} + \hat J_{2R})\big) + \frac{2}{\epsilon}\big(\hat N_L + (1-w\epsilon)(\hat J_{1L} + \hat J_{2L})\big) + \\ &\quad + \frac{\alpha'_{\rm eff}}{\epsilon^2 \bar R^2}\big(m - (1-\epsilon)(\hat J_1 + \hat J_2)\big)^2 + \frac{w^2 \bar R^2}{\alpha'_{\rm eff}} \,. \end{aligned} \tag{4.6}$$

Therefore the only $w \neq 0$ states which have finite mass in the limit $\epsilon \to 0$ with $\alpha'_{\rm eff}$, $\bar R$ fixed are those with the following quantum numbers

$$\begin{aligned} \hat J_{1R} + \hat J_{2R} &= \hat N_R \,, \qquad \hat J_{1L} + \hat J_{2L} = -\hat N_L \,, \\ m = \hat J_1 + \hat J_2 &= \hat N_R - \hat N_L \,, \qquad w = 1 \,. \end{aligned} \tag{4.7}$$

They have finite masses given by

$$\alpha'_{\rm eff} M^2 = 2(\hat N_R + \hat N_L) + \frac{\alpha'_{\rm eff}}{\bar R^2}\big(\hat N_R - \hat N_L\big)^2 + \frac{\bar R^2}{\alpha'_{\rm eff}} \,. \tag{4.8}$$

Now consider the neutral sector with $w = 0$. In this case the spectrum is the same as the free superstring spectrum in flat Minkowski spacetime [10]. Therefore

$$\alpha'_{\text{eff}} M^2 = \frac{2}{\epsilon}(\hat{N}_R + \hat{N}_L)\,, \qquad \hat{N}_R = \hat{N}_L\,. \tag{4.9}$$

In the limit $\epsilon \to 0$, all string excitations decouple, leaving only the massless supergravity multiplet $\hat{N}_R = \hat{N}_L = 0$.

Thus the full theory after the limit is type-II supergravity coupled to the infinite number of states (4.7) of the leading Regge trajectory with masses given by (4.8).

To conclude, we have seen that there is a limit in superstring theory in which all string excitations can be decoupled except for certain string states lying on the leading Regge trajectory. This implies a great simplification of the theory. An interesting problem is to understand if the resulting theory can be described in terms of a (Lorentz invariant) quantum field theory in 3+1 dimensions. It would be interesting to see if it is possible to implement this limit at the level of string sigma model, and to see how to formulate interactions in the resulting model without having to go through a limiting process of string-theory scattering amplitudes. It would also be interesting to see if one can take analogous limits in Dp-branes.

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